Federal Competition For Advanced Students, Part 2 1986

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by parmenides51, moldovan

–	Day 1
1	Show that a square can be inscribed in any regular polygon.
2	For $s,t \in \mathbb{N}$, consider the set $M = \{(x,y) \in \mathbb{N}^2 \mid 1 \leq x \leq s, 1 \leq y \leq t\}$. Find the number of rhombi with the vertices in M and the diagonals parallel to the coordinate axes.
3	Find all possible values of x_0 and x_1 such that the sequence defined by: $x_{n+1} = \frac{x_{n-1}x_n}{3x_{n-1}-2x_n}$ for $n \geq 1$ contains infinitely many natural numbers.
–	Day 2
4	Find the largest n for which there is a natural number N with n decimal digits which are all different such that $n!$ divides N. Furthermore, for this largest n find all possible numbers N.
5	Show that for every convex n-gon $(n \geq 4)$, the arithmetic mean of the lengths of its sides is less than the arithmetic mean of the lengths of all its diagonals.
6	Given a positive integer n, find all functions $F : \mathbb{N} \to \mathbb{R}$ such that $F(x+y) = F(xy-n)$ whenever $x,y \in \mathbb{N}$ satisfy $xy > n$.