Federal Competition For Advanced Students, Part 2 1985

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–	Day 1
1	Determine all quadruples (a, b, c, d) of nonnegative integers satisfying: $a^2 + b^2 + c^2 + d^2 = a^2b^2c^2$.
2	For $n \in \mathbb{N}$, let $f(n) = 1^n + 2^{n-1} + 3^{n-2} + ... + n^1$. Determine the minimum value of: $\frac{f(n+1)}{f(n)}$.
3	A line meets the lines containing sides BC, CA, AB of a triangle ABC at A_1, B_1, C_1, respectively. Points A_2, B_2, C_2 are symmetric to A_1, B_1, C_1 with respect to the midpoints of BC, CA, AB, respectively. Prove that A_2, B_2, and C_2 are collinear.
–	Day 2
4	Find all natural numbers n such that the equation: $a_{n+1}x^2 - 2x\sqrt{a_1^2 + a_2^2 + ... + a_{n+1}^2} + a_1 + a_2 + ... + a_n = 0$ has real solutions for all real numbers $a_1, a_2, ..., a_{n+1}$.
5	A sequence (a_n) of positive integers satisfies: $a_n = \sqrt{\frac{a_{n-1}^2 + a_{n+1}^2}{2}}$ for all $n \geq 1$. Prove that this sequence is constant.
6	Find all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying: $x^2 f(x) + f(1 - x) = 2x - x^4$ for all $x \in \mathbb{R}$.