

Federal Competition For Advanced Students, Part 1 1991

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- 1 Suppose that a, b , and $\sqrt[3]{a} + \sqrt[3]{b}$ are rational numbers. Prove that $\sqrt[3]{a}$ and $\sqrt[3]{b}$ are also rational.

 - 2 Solve in real numbers the equation:
$$\frac{1}{x} + \frac{1}{x+2} - \frac{1}{x+4} - \frac{1}{x+6} - \frac{1}{x+8} - \frac{1}{x+10} + \frac{1}{x+12} + \frac{1}{x+14} = 0.$$

 - 3 Find the number of squares in the sequence given by $a_0 = 91$ and $a_{n+1} = 10a_n + (-1)^n$ for $n \geq 0$.

 - 4 Let AB be a chord of a circle k of radius r , with $AB = c$. (a) Construct the triangle ABC with C on k in which a median from A or B is of a given length d . (b) For which c and d is this triangle unique?
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