Federal Competition For Advanced Students, Part 1 1989

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1	Natural numbers $a \leq b \leq c \leq d$ satisfy $a+b+c+d=30$. Find the maximum value of the product $P=abcd$.
2	If a and b are nonnegative real numbers with $a^2+b^2=4$, show that: $\frac{ab}{a+b+2} \leq \sqrt{2}-1$ and determine when equality occurs.
3	Let a be a real number. Prove that if the equation $x^2-ax+a=0$ has two real roots x_1 and x_2, then: $x_1^2+x_2^2 \geq 2(x_1+x_2)$.
4	Prove that for any triangle each exradius is less than four times the circumradius.

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