Dutch Mathematical Olympiad 1997

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by parmenides51

1 For each positive integer n we define $f(n)$ as the product of the sum of the digits of n with n itself.
Examples: $f(19) = (1 + 9) \times 19 = 190$, $f(97) = (9 + 7) \times 97 = 1552$.
Show that there is no number n with $f(n) = 19091997$.

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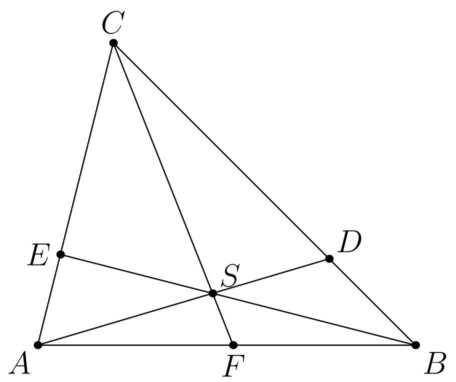
2 The lines AD, BE and CF intersect in S within a triangle ABC .
It is given that $AS : DS = 3 : 2$ and $BS : ES = 4 : 3$. Determine the ratio $CS : FS$.



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3 a. View the second-degree quadratic equation $x^2+?x+? = 0$
Two players successively put an integer each at the location of a question mark. Show that the second player can always ensure that the quadratic gets two integer solutions.
Note: we say that the quadratic also has two integer solutions, even when they are equal (for example if they are both equal to 3).

b.View the third-degree equation $x^3+?x^2+?x+? = 0$
Three players successively put an integer each at the location of a question mark. The equation appears to have three integer (possibly again the same) solutions. It is given that two players each put a 3 in the place of a question mark. What number did the third player put? Determine that number and the place where it is placed and prove that only one number is possible.

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4 We look at an octahedron, a regular octahedron, having painted one of the side surfaces red and the other seven surfaces blue. We throw the octahedron like a die. The surface that comes up is painted: if it is red it is painted blue and if it is blue it is painted red. Then we throw the octahedron again and paint it again according to the above rule. In total we throw the octahedron 10 times.

How many different octahedra can we get after finishing the 10th time?

Two octahedra are different if they cannot be converted into each other by rotation.

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5 Given is a triangle ABC and a point K within the triangle. The point K is mirrored in the sides of the triangle: P, Q and R are the mirrorings of K in AB, BC and CA, respectively . M is the center of the circle passing through the vertices of triangle PQR. M is mirrored again in the sides of triangle ABC: P', Q' and R' are the mirror of M in AB respectively, BC and CA.
a. Prove that K is the center of the circle passing through the vertices of triangle $P'Q'R'$.
b. Where should you choose K within triangle ABC so that M and K coincide? Prove your answer.
