## AoPS Community

## Dutch Mathematical Olympiad 1996

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1 How many different (non similar) triangles are there whose angles have an integer number of degrees?

2 Investigate whether for two positive integers $m$ and $n$ the numbers $m^{2}+n$ and $n^{2}+m$ can be both squares of integers.

3 What is the largest number of horses that you can put on a chessboard without there being two horses that can beat each other?
a. Describe an arrangement with that maximum number.
b. Prove that a larger number is not possible.
(A chessboard consists of $8 \times 8$ spaces and a horse jumps from one field to another field according to the line "two squares vertically and one squared horizontally" or "one square vertically and two squares horizontally")

$4 \quad$ A line $l$ intersects the segment $A B$ perpendicular to $C$. Three circles are drawn successively with $A B, A C$ and $B C$ as the diameter. The largest circle intersects $l$ in $D$. The segments $D A$ and $D B$ still intersect the two smaller circles in $E$ and $F$.
a. Prove that quadrilateral $C F D E$ is a rectangle.
b. Prove that the line through $E$ and $F$ touches the circles with diameters $A C$ and $B C$ in $E$ and $F$.

$5 \quad$ For the positive integers $x, y$ and $z$ apply $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$.
Prove that if the three numbers $x, y$, and $z$ have no common divisor greater than $1, x+y$ is the square of an integer.

