

**ITAMO 1993**

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- 1 Let be given points  $A, B, C$  on a line, with  $C$  between  $A$  and  $B$ . Three semicircles with diameters  $AC, BC, AB$  are drawn on the same side of line  $ABC$ . The perpendicular to  $AB$  at  $C$  meets the circle with diameter  $AB$  at  $H$ . Given that  $CH = \sqrt{2}$ , compute the area of the region bounded by the three semicircles.

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- 2 Find all pairs  $(p, q)$  of positive primes such that the equation  $3x^2 - px + q = 0$  has two distinct rational roots.

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- 3 Consider an infinite chessboard whose rows and columns are indexed by positive integers. At most one coin can be put on any cell of the chessboard. Let be given two arbitrary sequences  $(a_n)$  and  $(b_n)$  of positive integers ( $n \in \mathbb{N}$ ). Assuming that infinitely many coins are available, prove that they can be arranged on the chessboard so that there are  $a_n$  coins in the  $n$ -th row and  $b_n$  coins in the  $n$ -th column for all  $n$ .

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- 4 Let  $P$  be a point in the plane of a triangle  $ABC$ , different from its circumcenter. Prove that the triangle whose vertices are the projections of  $P$  on the perpendicular bisectors of the sides of  $ABC$ , is similar to  $ABC$ .

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- 5 Prove the following inequality for any positive real numbers  $a, b, c$  not exceeding 1  $a^2b + b^2c + c^2a + 1 \geq a^2 + b^2 + c^2$

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- 6 A unit cube  $C$  is rotated around one of its diagonals for the angle  $\pi/3$  to form a cube  $C'$ . Find the volume of the intersection of  $C$  and  $C'$ .

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