ITAMO 1989

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by parmenides51

1 Determine whether the equation $x^2 + xy + y^2 = 2$ has a solution (x, y) in rational numbers.

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2 There are 30 men with their 30 wives sitting at a round table. Show that there always exist two men who are on the same distance from their wives. (The seats are arranged at vertices of a regular polygon.)

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3 Prove that, for every tetrahedron $ABCD$, there exists a unique point P in the interior of the tetrahedron such that the tetrahedra $PABC, PABD, PACD, PBCD$ have equal volumes.

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4 Points A, M, B, C, D are given on a circle in this order such that A and B are equidistant from M. Lines MD and AC intersect at E and lines MC and BD intersect at F. Prove that the quadrilateral $CDEF$ is inscridable in a circle.

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5 A fair coin is repeatedly tossed. We receive one marker for every head and two markers for every tail. We win the game if, at some moment, we possess exactly 100 markers. Is the probability of winning the game greater than, equal to, or less than $2/3$?

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6 Given a real number α, a function f is defined on pairs of nonnegative integers by $f(0, 0) = 1, f(m, 0) = f(0, m) = 0$ for $m > 0$, $f(m, n) = \alpha f(m, n-1) + (1-\alpha) f(m-1, n-1)$ for $m, n > 0$. Find the values of α such that $|f(m, n)| < 1989$ holds for any integers $m, n \geq 0$.
