Czech And Slovak Mathematical Olympiad, Round III, Category A 2010

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by parmenides51

1 Determine all pairs of integers a, b for which they apply $4^a + 4a^2 + 4 = b^2$.

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2 A circular target with a radius of 12 cm was hit by 19 shots. Prove that the distance between two hits is less than 7 cm.

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3 Rumburak kidnapped 31 members of party A , 28 members of party B, 23 members of party C, 19 members of Party D and each of them in a separate cell. After work out occasionally they could walk in the yard and talk. Once three people started to talk to each other members of three different parties, Rumburak re-registered them to the fourth party as a punishment.(They never talked to each other more than three kidnapped.)
a) Could it be that after some time all were abducted by members of one party? Which?
b) Determine all four positive integers of which the sum is 101 and which as the numbers of kidnapped members of the four parties allow the Rumburaks all of them became members of one party over time.

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4 A circle k is given with a non-diameter chord AC. On the tangent at point A select point $X \neq A$ and mark D the intersection of the circle k with the interior of the line XC (if any). Let B a point in circle k such that quadrilateral $ABCD$ is a trapezoid . Determine the set of intersections of lines BC and AD belonging to all such trapezoids.

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5 On the board are written numbers $1, 2, ..., 33$. In one step we select two numbers written on the product of which is the square of the natural number, we wipe off the two chosen numbers and write the square root of their product on the board. This way we continue to the board only the numbers remain so that the product of neither of them is a square. (In one we can also wipe out two identical numbers and replace them with the same number.) Prove that at least 16 numbers remain on the board.

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6 Find the minimum of the expression $\frac{a+b+c}{2} - \frac{[a,b]+[b,c]+[c,a]}{a+b+c}$ where the variables a, b, c are any integers greater than 1 and $[x, y]$ denotes the least common multiple of numbers x, y.
