Czech-Polish-Slovak Junior Match 2018

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by parmenides51

–	Individual
1	Are there four real numbers a, b, c, d for every three positive real numbers x, y, z with the property $ad + bc = x$, $ac + bd = y$, $ab + cd = z$ and one of the numbers a, b, c, d is equal to the sum of the other three?
2	A convex hexagon $ABCDEF$ is given whose sides AB and DE are parallel. Each of the diagonals AD, BE, CF divides this hexagon into two quadrilaterals of equal perimeters. Show that these three diagonals intersect at one point.
3	The teacher gave each of her 37 students 36 pencils in different colors. It turned out that each pair of students received exactly one pencil of the same color. Determine the smallest possible number of different colors of pencils distributed.
4	Determine the smallest positive integer A with an odd number of digits and this property, that both A and the number B created by removing the middle digit of the number A are divisible by 2018.
5	An acute triangle ABC is given in which $AB < AC$. Point E lies on the AC side of the triangle, with $AB = AE$. The segment AD is the diameter of the circumcircle of the triangle ABC, and point S is the center of this arc BC of this circle to which point A does not belong. Point F is symmetric of point D wrt S. Prove that lines FE and AC are perpendicular.
–	Team
1	For natural numbers a, bc it holds that $(a + b + c)^2 \vert ab(a + b) + bc(b + c) + ca(c + a) + 3abc$. Prove that $(a + b + c) \vert (a - b)^2 + (b - c)^2 + (c - a)^2$
2	Given a right triangle ABC with the hypotenuse AB. Let K be any interior point of triangle ABC and points L, M are symmetric of point K wrt lines BC, AC respectively. Specify all possible values for S_{ABLM}/S_{ABC}, where $S_{XY...Z}$ indicates the area of the polygon $XY...Z$.
3	Calculate all real numbers r with the following properties: If real numbers a, b, c satisfy the inequality$\vert ax^2 + bx + c\vert \le 1$ for each $x \in [-1, 1]$, then they also satisfy the inequality $\vert cx^2 + bx + a\vert \le r$ for each $x \in [-1, 1]$.
4	A line passing through the center M of the equilateral triangle ABC intersects sides BC and CA, respectively, in points D and E. Circumcircles of triangle AEM and BDM intersects,

besides point M, also at point P. Prove that the center of circumcircle of triangle DEP lies on the perpendicular bisector of the segment AB.

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5 There are $2n$ people ($n \geq 2$) sitting around the round table, with each person getting to know both with his neighbors and exactly opposite him sits a person he does not know. Prove that people can rearrange in such a way that everyone knows one of their two neighbors.

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6 Positive real numbers a, b are such that $a^3 + b^3 = 2$.
Show that that $\frac{1}{a} + \frac{1}{b} \geq 2(a^2 - a + 1)(b^2 - b + 1)$.
