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1 Two circles α and β intersect at points P and Q. The lines connecting a point R on β with P and Q intersect α again at S and T respectively. Prove that ST is parallel to the line tangent to β at R.

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2 Determine the general term of the sequence (a_n) given by $a_0 = \alpha > 0$ and $a_{n+1} = \frac{a_n}{1+a_n}$.

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3 Two numbers are randomly selected from interval $I = [0, 1]$.
Given $\alpha \in I$, what is the probability that the smaller of the two numbers does not exceed α?

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4 Prove that a circle centered at point $(\sqrt{2}, \sqrt{3})$ in the cartesian plane passes through at most one point with integer coordinates.

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5 Given an acute triangle T with sides a, b, c, find the tetrahedra with base T whose all faces are acute triangles of the same area.

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6 Show that for any positive integer n there exists an integer $m > 1$ such that $(\sqrt{2}-1)^n = \sqrt{m} - \sqrt{m-1}$.

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7 On a long enough highway, a passenger in a bus observes the traffic. He notes that, during an hour, the bus going with a constant velocity overpasses a cars and gets overpassed by b cars, while c cars pass in the opposite direction. Assuming that the traffic is the same in both directions, is it possible to determine the number of cars that pass along the highway per hour? (You may assume that the velocity of a car can take only two values.)
