

Federal Competition For Advanced Students 2014

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- 1 Determine all real numbers x and y such that $x^2 + x = y^3 - y$, $y^2 + y = x^3 - x$

- 2 We call a set of squares with sides parallel to the coordinate axes and vertices with integer coordinates friendly if any two of them have exactly two points in common. We consider friendly sets in which each of the squares has sides of length n . Determine the largest possible number of squares in such a friendly set.

- 3 Let a_n be a sequence defined by some a_0 and the recursion $a_{n+1} = a_n + 2 \cdot 3^n$ for $n \geq 0$. Determine all rational values of a_0 such that a_k^j / a_j^k is an integer for all integers j and k with $0 < j < k$.

- 4 We are given a right-angled triangle MNP with right angle in P . Let k_M be the circle with center M and radius MP , and let k_N be the circle with center N and radius NP . Let A and B be the common points of k_M and the line MN , and let C and D be the common points of k_N and the line MN with C between A and B . Prove that the line PC bisects the angle $\angle APB$.