Czech And Slovak Mathematical Olympiad, Round III, Category A 2016

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1   Let $p > 3$ be a prime number. Determine the number of all ordered sixes (a, b, c, d, e, f) of positive integers whose sum is $3p$ and all fractions $\frac{a+b}{c+d}, \frac{b+c}{d+e}, \frac{c+d}{e+f}, \frac{d+e}{f+a}, \frac{e+f}{a+b}$ have integer values.

2   Let us denote successively r and r_a the radii of the inscribed circle and the exscribed circle wrt to side BC of triangle ABC. Prove that if it is true that $r + r_a = |BC|$, then the triangle ABC is a right one

3   Mathematical clubs are popular among the inhabitants of the same city. Every two of them they have at least one member in common. Prove that we can give the people of the city compasses and rulers so that only one inhabitant gets both, while each club will to have both a ruler and a compass at the full participation of its members.

4   For positive numbers a, b, c holds $(a + c)(b^2 + ac) = 4a$.
Determine the maximum value of $b + c$ and find all triplets of numbers (a, b, c) for which expression takes this value

5   In the triangle $ABC, |BC| = 1$ and there is exactly one point D on the side BC such that $|DA|^2 = |DB| \cdot |DC|$. Determine all possible values of the perimeter of the triangle ABC.

6   We put a figure of a king on some 6×6 chessboard. It can in one thrust jump either vertically or horizontally. The length of this jump is alternately one and two squares, whereby a jump of one (i.e. to the adjacent square) of the piece begins. Decide whether you can choose the starting position of the pieces so that after a suitable sequence 35 jumps visited each box of the chessboard just once.