Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 1993

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by parmenides51

1a Let $ABCD$ be a convex quadrilateral and $A', B'C', D'$ be the midpoints of AB, BC, CD, DA, respectively. Let a, b, c, d denote the areas of quadrilaterals into which lines $A'C'$ and $B'D'$ divide the quadrilateral $ABCD$ (where a corresponds to vertex A etc.). Prove that $a+c = b+d$.

1b Given a triangle with sides of lengths a, b, c, prove that $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} < 2$.

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2 If a, b, c, d are real numbers with $b < c < d$, prove that $(a+b+c+d)^2 > 8(ac+bd)$.

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3 The Fermat-numbers are defined by $F_n = 2^{2^n} + 1$ for $n \in N$.
(a) Prove that $F_n = F_{n-1}F_{n-2}....F_1F_0 + 2$ for $n > 0$.
(b) Prove that any two different Fermat numbers are coprime

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4 Each of the 8 vertices of a given cube is given a value 1 or -1.
Each of the 6 faces is given the value of product of its four vertices.
Let A be the sum of all the 14 values. Which are the possible values of A?
