**Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 1996**

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by parmenides51

**1** Let $S$ be a circle with center $C$ and radius $r$, and let $P \neq C$ be an arbitrary point.
A line $\ell$ through $P$ intersects the circle in $X$ and $Y$. Let $Z$ be the midpoint of $XY$.
Prove that the points $Z$, as $\ell$ varies, describe a circle. Find the center and radius of this circle.

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**2** Prove that $[\sqrt{n}+\sqrt{n+1}]=[\sqrt{4n+1}]$ for all $n \in N$.

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**3** Per and Kari each have $n$ pieces of paper. They both write down the numbers from $1$ to $2n$ in an arbitrary order, one number on each side. Afterwards, they place the pieces of paper on a table showing one side. Prove that they can always place them so that all the numbers from $1$ to $2n$ are visible at once.

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**4** Let $f : N \to N$ be a function such that $f(f(1995)) = 95$, $f(xy) = f(x)f(y)$ and $f(x) \leq x$ for all $x, y$.
Find all possible values of $f(1995)$.

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