Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 1999

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by parmenides51

1a Find a function f such that $f(t^2+t+1)=t$ for all real $t\geq 0$

1b If a,b,c,d,e are real numbers, prove the inequality $a^2+b^2+c^2+d^2+e^2\geq a(b+c+d+e)$.

2a Find all integers m and n such that $2m^2+n^2=2mn+3n$

2b If a,b,c are positive integers such that $b|a^3, c|b^3$ and $a|c^3$, prove that $abc|(a+b+c)^{13}$

3 An isosceles triangle ABC with $AB=AC$ and $\angle A=30^o$ is inscribed in a circle with center O. Point D lies on the shorter arc AC so that $\angle DOC=30^o$, and point G lies on the shorter arc AB so that $DG=AC$ and $AG<BG$. The line BG intersects AC and AB at E and F, respectively.
(a) Prove that triangle AFG is equilateral.
(b) Find the ratio between the areas of triangles AFE and ABC.

4 For every nonempty subset R of $S=\{1,2,...,10\}$, we define the alternating sum $A(R)$ as follows:
If $r_1,r_2,...,r_k$ are the elements of R in the increasing order, then $A(R)=r_k-r_{k-1}+r_{k-2}-...+(-1)^{k-1}r_1$.
(a) Is it possible to partition S into two sets having the same alternating sum?
(b) Determine the sum $\sum_R A(R)$, where R runs over all nonempty subsets of S.