Czech And Slovak Mathematical Olympiad, Round III, Category A 2001

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by parmenides51

1 Determine all polynomials P such that for every real number x, $P(x)^2+P(-x)=P(x^2)+P(x)$

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2 Given a triangle PQX in the plane, with $PQ=3, PX=2.6$ and $QX=3.8$. Construct a right-angled triangle ABC such that the incircle of $\triangle ABC$ touches AB at P and BC at Q, and point X lies on the line AC.

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3 Find all triples of real numbers (a,b,c) for which the set of solutions x of $\sqrt{2x^2+ax+b}>x-c$ is the set $(-\infty,0]\cup(1,\infty)$.

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4 In a certain language there are n letters. A sequence of letters is a word, if there are no two equal letters between two other equal letters. Find the number of words of the maximum length.

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5 A sheet of paper has the shape of an isosceles trapezoid $C_1AB_2C_2$ with the shorter base B_2C_2. The foot of the perpendicular from the midpoint D of C_1C_2 to AC_1 is denoted by B_1. Suppose that upon folding the paper along DB_1, AD and AC_1 points C_1,C_2 become a single point C and points B_1,B_2 become a point B. The area of the tetrahedron $ABCD$ is 64 cm^2 . Find the sides of the initial trapezoid.

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6 Let be given natural numbers $a_1,a_2,...,a_n$ and a function $f:Z\to R$ such that $f(x)=1$ for all integers $x<0$ and $f(x)=1-f(x-a_1)f(x-a_2)...f(x-a_n)$ for all integers $x\geq 0$. Prove that there exist natural numbers s and t such that for all integers $x>s$ it holds that $f(x+t)=f(x)$.
