Czech And Slovak Mathematical Olympiad, Round III, Category A 2012

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by parmenides51

1 Find all integers for which n is n^4-3n^2+9 prime

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2 Find out the maximum possible area of the triangle ABC whose medians have lengths satisfying inequalities $m_a \le 2, m_b \le 3, m_c \le 4$.

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3 Prove that there are two numbers u and v, between any 101 real numbers that apply $100|u-v|\cdot|1-uv| \le (1+u^2)(1+v^2)$

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4 Inside the parallelogram $ABCD$ is a point X. Make a line that passes through point X and divides the parallelogram into two parts whose areas differ from each other the most.

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5 In a group of 90 children each has at least 30 friends (friendship is mutual). Prove that they can be divided into three 30-member groups so that each child has its own a group of at least one friend.

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6 In the set of real numbers solve the system of equations $x^4+y^2+4=5yz$ $y^4+z^2+4=5zx$ $z^4+x^2+4=5xy$
