Czech And Slovak Mathematical Olympiad, Round III, Category A 2013

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by parmenides51

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1 Find all pairs of integers a, b for which equality holds $\frac{a^2+1}{2b^2-3}=\frac{a-1}{2b-1}$

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2 Each of the thieves in the n-member party ($n \ge 3$) charged a certain number of coins. All the coins were $100n$. Thieves decided to share their prey as follows: at each step, one of the bandits puts one coin to the other two. Find them all natural numbers $n \ge 3$ for which after a finite number of steps each outlaw can have 100 coins no matter how many coins each thug has charged.

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3 In the parallelolgram ABCD with the center S, let O be the center of the circle of the inscribed triangle ABD and let T be the touch point with the diagonal BD. Prove that the lines OS and CT are parallel.

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4 On the board is written in decimal the integer positive number N. If it is not a single digit number, wipe its last digit c and replace the number m that remains on the board with a number $m - 3c$. (For example, if $N = 1,204$ on the board, $120 - 3 \cdot 4 = 108$.) Find all the natural numbers N, by repeating the adjustment described eventually we get the number 0.

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5 Given the parallelogram $ABCD$ such that the feet K, L of the perpendiculars from point D on the sides AB, BC respectively are internal points. Prove that $KL \parallel AC$ when $|\angle BCA| + |\angle ABD| = |\angle BDA| + |\angle ACD|$.

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6 Find all positive real numbers p such that $\sqrt{a^2+pb^2}+\sqrt{b^2+pa^2} \ge a+b+(p-1)\sqrt{ab}$ holds for any pair of positive real numbers a, b.
