

Croatia Team Selection Test 2002

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- 1 In a certain language there are n letters. A sequence of letters is a word, if there are no two equal letters between two other equal letters. Find the number of words of the maximum length.

- 2 A quadrilateral $ABCD$ is circumscribed about a circle. Lines AC and DC meet at point E and lines DA and BC meet at F , where B is between A and E and between C and F . Let I_1, I_2 and I_3 be the incenters of triangles AFB, BEC and ABC , respectively. The line I_1I_3 intersects EA at K and ED at L , whereas the line I_2I_3 intersects FC at M and FD at N . Prove that $EK = EL$ if and only if $FM = FN$.

- 3 Prove that if n is a natural number such that $1 + 2^n + 4^n$ is prime then $n = 3^k$ for some $k \in N_0$.