Italy TST 1998

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1   A real number α is given. Find all functions $f : R^+ \to R^+$ satisfying $\alpha x^2 f\left(\frac{1}{x}\right) + f(x) = \frac{x}{x+1}$ for all $x > 0$.

2   In a triangle ABC, points H, M, L are the feet of the altitude from C, the median from A, and the angle bisector from B, respectively. Show that if triangle HML is equilateral, then so is triangle ABC.

3   New license plates consist of two letters, three digits, and two letters (from the English alphabet of 26 letters). What is the largest possible number of such license plates if it is required that every two of them differ at no less than two positions?

4   Find all polynomials $P(x) = x^n + a_1 x^{n-1} + ... + a_n$ whose zeros (with their multiplicities) are exactly $a_1, a_2, ..., a_n$.