

Italy TST 1993

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- 1 Let x_1, x_2, \dots, x_n ($n \geq 2$) be positive numbers with the sum 1. Prove that

$$\sum_{i=1}^n \frac{1}{\sqrt{1-x_i}} \geq n\sqrt{\frac{n}{n-1}}$$

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- 2 Suppose that p, q are prime numbers such that $\sqrt{p^2 + 7pq + q^2} + \sqrt{p^2 + 14pq + q^2}$ is an integer. Show that $p = q$.
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- 3 Let ABC be an isosceles triangle with base AB and D be a point on side AB such that the incircle of triangle ACD is congruent to the excircle of triangle DCB across C . Prove that the diameter of each of these circles equals half the altitude of $\triangle ABC$ from A .
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- 4 An $m \times n$ chessboard with $m, n \geq 2$ is given. Some dominoes are placed on the chessboard so that the following conditions are satisfied:
(i) Each domino occupies two adjacent squares of the chessboard,
(ii) It is not possible to put another domino onto the chessboard without overlapping,
(iii) It is not possible to slide a domino horizontally or vertically without overlapping.
Prove that the number of squares that are not covered by a domino is less than $\frac{1}{5}mn$.
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