Italy TST 1994

www.artofproblemsolving.com/community/c1075281

by parmenides51

1   Given a circle γ and a point P inside it, find the maximum and minimum value of the sum of the lengths of two perpendicular chords of γ passing through P.

2   Find all prime numbers p for which $\frac{2^{p-1}-1}{p}$ is a perfect square.

3   Find all functions $f : R \to R$ satisfying the condition $f(x - f(y)) = 1 + x - y$ for all $x, y \in R$.

4   Let X be a set of n elements and k be a positive integer.
Consider the family S_k of all k-tuples $(E_1, ..., E_k)$ with $E_i \subseteq X$ for each i.
Evaluate the sums $\sum_{(E_1,...,E_k)\in S_k} |E_1 \cap ... \cap E_k|$ and $\sum_{(E_1,...,E_k)\in S_k} |E_1 \cup ... \cup E_k|$