Romania Team Selection Test 1993

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–	BMO TST
1	Consider the sequence $z_n = (1+i)(2+i)...(n+i)$. Prove that the sequence $Im\ z_n$ contains infinitely many positive and infinitely many negative numbers.
2	Let ABC be a triangle inscribed in the circle $\mathcal{C}(O,R)$ and circumscribed to the circle $\mathcal{C}(L,r)$. Denote $d = \dfrac{Rr}{R+r}$. Show that there exists a triangle DEF such that for any interior point M in ABC there exists a point X on the sides of DEF such that $MX \le d$. *Dan Brnzei*
3	Show that the set $\{1,2,....,2^n\}$ can be partitioned in two classes, none of which contains an arithmetic progression of length $2n$.
4	Prove that the equation $(x+y)^n = x^m + y^m$ has a unique solution in integers with $x > y > 0$ and $m, n > 1$.
–	IMO TST
–	Day 1
1	Find max. numbers A wich is true ineq.: $\frac{x}{\sqrt{y^2+z^2}} + \frac{y}{\sqrt{x^2+z^2}} + \frac{z}{\sqrt{x^2+y^2}} \ge A$ x, y, z are positve reals numberes! :wink:
2	$x^2 + y^2 + z^2 = 1993$ then prove $x+y+z$ can't be a perfect square:
3	Suppose that each of the diagonals AD, BE, CF divides the hexagon $ABCDEF$ into two parts of the same area and perimeter. Does the hexagon necessarily have a center of symmetry?
4	Let Y be the family of all subsets of $X = \{1,2,...,n\}$ ($n > 1$) and let $f : Y \to X$ be an arbitrary mapping. Prove that there exist distinct subsets A, B of X such that $f(A) = f(B) = maxA\triangle B$, where $A\triangle B = (A-B)\cup(B-A)$.
–	Day 2

1 Let $f : R^+ \to R$ be a strictly increasing function such that $f\left(\frac{x+y}{2}\right) < \frac{f(x)+f(y)}{2}$ for all $x, y > 0$. Prove that the sequence $a_n = f(n)$ ($n \in N$) does not contain an infinite arithmetic progression.

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2 For coprime integers $m > n > 1$ consider the polynomials $f(x) = x^{m+n} - x^{m+1} - x + 1$ and $g(x) = x^{m+n} + x^{n+1} - x + 1$. If f and g have a common divisor of degree greater than 1, find this divisor.

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3 Find all integers $n > 1$ for which there is a set B of n points in the plane such that for any $A \in B$ there are three points $X, Y, Z \in B$ with $AX = AY = AZ = 1$.

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4 For each integer $n > 3$ find all quadruples (n_1, n_2, n_3, n_4) of positive integers with $n_1 + n_2 + n_3 + n_4 = n$ which maximize the expression

$$\frac{n!}{n_1!n_2!n_3!n_4!}2^{\binom{n_1}{2}+\binom{n_2}{2}+\binom{n_3}{2}+\binom{n_4}{2}+n_1n_2+n_2n_3+n_3n_4}$$

– Day 3

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1 Define the sequence (x_n) as follows: the first term is 1, the next two are $2, 4$, the next three are $5, 7, 9$, the next four are $10, 12, 14, 16$, and so on. Express x_n as a function of n.

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2 Suppose that D, E, F are points on sides BC, CA, AB of a triangle ABC respectively such that $BD = CE = AF$ and $\angle BAD = \angle CBE = \angle ACF$.Prove that the triangle ABC is equilateral.

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3 Let $p \geq 5$ be a prime number.Prove that for any partition of the set $P = \{1, 2, 3, ..., p-1\}$ in 3 subsets there exists numbers x, y, z each belonging to a distinct subset,such that $x + y \equiv z(modp)$

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