

**Switzerland Team Selection Test 1997**

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by Medjl

1. A finite sequence of integers  $a_0, a_1, \dots, a_n$  is called quadratic if  $|a_k - a_{k-1}| = k^2$  for  $n \geq k \geq 1$ .
  - (a) Prove that for any two integers  $b$  and  $c$ , there exist a natural number  $n$  and a quadratic sequence with  $a_0 = b$  and  $a_n = c$ .
  - (b) Find the smallest natural number  $n$  for which there exists a quadratic sequence with  $a_0 = 0$  and  $a_n = 1997$

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2. Let ABCD be a convex quadrilateral. Find the necessary and sufficient condition for the existence of point P inside the quadrilateral such that the triangles ABP, BCP, CDP, DAP have the same area

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3. A 66 square has been tiled by 18 dominoes. Show that there exists a line that divides the square into two parts, each of which is also tiled by dominoes

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4. Let  $v$  and  $w$  be two randomly chosen roots of the equation  $z^{1997} - 1 = 0$  (all roots are equiprobable). Find the probability that  $\sqrt{2 + \sqrt{3}} \leq |u + w|$