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Art of Problem Solving

Switzerland Team Selection Test 2002

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–	Day 1
1	In space are given 24 points, no three of which are collinear. Suppose that there are exactly 2002 planes determined by three of these points. Prove that there is a plane containing at least six points.
2	A point$O$ inside a parallelogram $ABCD$ satisfies $\angle AOB + \angle COD = \pi$. Prove that $\angle CBO = \angle CDO$.
3	Let d_1, d_2, d_3, d_4 be the four smallest divisors of a positive integer n (having at least four divisors). Find all n such that $d_1^2 + d_2^2 + d_3^2 + d_4^2 = n$.
4	A 7×7 square is divided into unit squares by lines parallel to its sides. Some Swiss crosses (obtained by removing corner unit squares from a square of side 3) are to be put on the large square, with the edges along division lines. Find the smallest number of unit squares that need to be marked in such a way that every cross covers at least one marked square.
5	Find all $f : R \to R$ such that (i) The set $\{\frac{f(x)}{x} \| x \in R - \{0\}\}$ is finite (ii) $f(x - 1 - f(x)) = f(x) - 1 - x$ for all x
–	Day 2
6	A sequence $x_1, x_2, x_3, ...$ has the following properties: (a) $1 = x_1 < x_2 < x_3 < ...$ (b) $x_{n+1} \le 2n$ for all $n \in N$. Prove that for each positive integer k there exist indices i and j such that $k = x_i - x_j$.
7	Let ABC be a triangle and P an exterior point in the plane of the triangle. Suppose the lines AP, BP, CP meet the sides BC, CA, AB (or extensions thereof) in D, E, F, respectively. Suppose further that the areas of triangles PBD, PCE, PAF are all equal. Prove that each of these areas is equal to the area of triangle ABC itself.
8	In a group of n people, every weekend someone organizes a party in which he invites all of his acquaintances. Those who meet at a party become acquainted. After each of the n people has organized a party, there still are two people not knowing each other. Show that these two will never get to know each other at such a party.

9 For each real number a and integer $n \geq 1$ prove the inequality $a^n + \frac{1}{a^n} - 2 \geq n^2 \left(a + \frac{1}{a} - 2\right)$ and find the cases of equality.

10 Given an integer $m \geq 2$, find the smallest integer $k > m$ such that for any partition of the set $\{m, m+1, .., k\}$ into two classes A and B at least one of the classes contains three numbers a, b, c (not necessarily distinct) such that $a^b = c$.