Regional Competition For Advanced Students 2013

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by parmenides51

1 For which integers between 2000 and 2010 (including) is the probability that a random divisor is smaller or equal 45 the largest?

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2 Determine all integers x satisfying

$$\left[\frac{x}{2}\right]\left[\frac{x}{3}\right]\left[\frac{x}{4}\right] = x^2.$$

($[y]$ is the largest integer which is not larger than y.)

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3 For non-negative real numbers a, b let $A(a,b)$ be their arithmetic mean and $G(a,b)$ their geometric mean. We consider the sequence $\langle a_n \rangle$ with $a_0 = 0, a_1 = 1$ and $a_{n+1} = A(A(a_{n-1}, a_n), G(a_{n-1}, a_n))$ for $n > 0$.
(a) Show that each $a_n = b_n^2$ is the square of a rational number (with $b_n \geq 0$).
(b) Show that the inequality $\left|b_n - \frac{2}{3}\right| < \frac{1}{2^n}$ holds for all $n > 0$.

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4 We call a pentagon *distinguished* if either all side lengths or all angles are equal. We call it *very distinguished* if in addition two of the other parts are equal. i.e. 5 sides and 2 angles or 2 sides and 5 angles.Show that every very distinguished pentagon has an axis of symmetry.
