Czech-Polish-Slovak Junior Match 2017

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by parmenides51

–	Individual
1	Find the largest integer $n \geq 3$ for which there is a n-digit number $\overline{a_1a_2a_3...a_n}$ with non-zero digits a_1, a_2 and a_n, which is divisible by $\overline{a_2a_3...a_n}$.
2	Given is the triangle ABC, with $\|AB\| + \|AC\| = 3 \cdot \|BC\|$. Let's denote D, E also points that $BCDA$ and $CBEA$ are parallelograms. On the sides AC and AB sides, F and G are selected respectively so that $\|AF\| = \|AG\| = \|BC\|$. Prove that the lines DF and EG intersect at the line segment BC
3	Prove that for all real numbers x, y holds $(x^2 + 1)(y^2 + 1) \geq 2(xy - 1)(x + y)$. For which integers x, y does equality occur?
4	Given is a right triangle ABC with perimeter 2, with $\angle B = 90^o$. Point S is the center of the excircle to the side AB of the triangle and H is the intersection of the heights of the triangle ABS . Determine the smallest possible length of the segment HS.
5	Each field of the table $(mn + 1) \times (mn + 1)$ contains a real number from the interval $[0, 1]$. The sum the numbers in each square section of the table with dimensions nxn is equal to n. Determine how big it can be sum of all numbers in the table.
–	Team
1	Decide if there are primes p, q, r such that $(p^2 + p)(q^2 + q)(r^2 + r)$ is a square of an integer.
2	Decide if exists a convex hexagon with all sides longer than 1 and all nine of its diagonals are less than 2 in length.
3	How many 8-digit numbers are $*2*0*1*7$, where four unknown numbers are replaced by stars, which are divisible by 7?
4	Bolek draw a trapezoid $ABCD$ trapezoid ($AB//CD$) on the board, with its midsegment line EF in it. Point intersection of his diagonal AC, BD denote by P, and his rectangular projection on line AB denote by Q. Lolek, wanting to tease Bolek, blotted from the board everything except segments EF and PQ. When Bolek saw it, wanted to complete the drawing and draw the original trapezoid, but did not know how to do it. Can you help Bolek?

5 In each square of the 100×100 square table, type $1, 2$, or 3. Consider all subtables $m \times n$, where $m = 2$ and $n = 2$. A subtable will be called *balanced* if it has in its corner boxes of four identical numbers boxes . For as large a number k prove, that we can always find k balanced subtables, of which no two overlap, i.e. do not have a common box.

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6 On the board are written 100 mutually different positive real numbers, such that for any three different numbers a, b, c is $a^2 + bc$ is an integer. Prove that for any two numbers x, y from the board , number $\frac{x}{y}$ is rational.
