**Czech And Slovak Mathematical Olympiad, Round III, Category A 1999**

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**1** We are allowed to put several brackets in the expression

$$\frac{29 : 28 : 27 : 26 : ... : 17 : 16}{15 : 14 : 13 : 12 : ... : 3 : 2}$$

(a) Find the smallest possible integer value we can obtain in that way.
(b) Find all possible integer values that can be obtained.

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**2** In a tetrahedron $ABCD, E$ and $F$ are the midpoints of the medians from $A$ and $D$. Find the ratio of the volumes of tetrahedra $BCEF$ and $ABCD$.

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**3** Show that there exists a triangle $ABC$ such that $a \neq b$ and $a + t_a = b + t_b$, where $t_a, t_b$ are the medians corresponding to $a, b$, respectively. Also prove that there exists a number $k$ such that every such triangle satisfies $a + t_a = b + t_b = k(a + b)$. Finally, find all possible ratios $a : b$ in such triangles.

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**4** In a certain language there are only two letters, $A$ and $B$. We know that
(i) There are no words of length $1$, and the only words of length $2$ are $AB$ and $BB$.
(ii) A segment of length $n > 2$ is a word if and only if it can be obtained from a word of length less than $n$ by replacing each letter $B$ by some (not necessarily the same) word.
Prove that the number of words of length $n$ is equal to $\frac{2^n+2\cdot(-1)^n}{3}$

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**5** Given an acute angle $APX$ in the plane, construct a square $ABCD$ such that $P$ lies on the side $BC$ and ray $PX$ meets $CD$ in a point $Q$ such that $AP$ bisects the angle $BAQ$.

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**6** Find all pairs of real numbers $a, b$ for which the system of equations $\frac{x+y}{x^2+y^2} = a$, $\frac{x^3+y^3}{x^2+y^2} = b$ has a real solution.

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