Czech And Slovak Mathematical Olympiad, Round III, Category A 1995

www.artofproblemsolving.com/community/c1078301

by parmenides51

1 Suppose that tetrahedron $ABCD$ satisfies $\angle BAC + \angle CAD + \angle DAB = \angle ABC + \angle CBD + \angle DBA = 180^o$. Prove that $CD \geq AB$.

---

2 Find the positive real numbers x, y for which $\frac{x+y}{2}$, $\sqrt{xy}$, $\frac{2xy}{x+y}$, $\sqrt{\frac{x^2+y^2}{2}}$ are integers whose sum is 66.

---

3 Five distinct points and five distinct lines are given in the plane. Prove that one can select two of the points and two of the lines so that none of the selected lines contains any of the selected points.

---

4 Do there exist 10000 ten-digit numbers divisible by 7, all of which can be obtained from one another by a reordering of their digits?

---

5 Let A, B be points on a circle k with center S such that $\angle ASB = 90^o$. Circles k_1 and k_2 are tangent to each other at Z and touch k at A and B respectively. Circle k_3 inside $\angle ASB$ is internally tangent to k at C and externally tangent to k_1 and k_2 at X and Y, respectively. Prove that $\angle XCY = 45^o$

---

6 Find all real parameters p for which the equation $x^3 - 2p(p + 1)x^2 + (p^4 + 4p^3 - 1)x - 3p^3 = 0$ has three distinct real roots which are sides of a right triangle.
