Czech And Slovak Mathematical Olympiad, Round III, Category A 1993

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by parmenides51

1 Find all natural numbers n for which $7n-1$ is divisible by $6n-1$

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2 In fields of a 19×19 table are written integers so that any two lying on neighboring fields differ at most by 2 (two fields are neighboring if they share a side). Find the greatest possible number of mutually different integers in such a table.

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3 Let AKL be a triangle such that $\angle ALK > 90^o + \angle LAK$. Construct an equilateral trapezoid $ABCD$ (i.e. with three sides equal) with $AB \perp CD$ such that K lies on the side BC, L on the diagonal AC and the lines AK and BL intersect at the circumcenter of the trapezoid.

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4 The sequence (a_n) of natural numbers is defined by $a_1 = 2$ and $a_n + 1$ equals the sum of tenth powers of the digits of a_n for all $n \geq 1$. Are there numbers which appear twice in the sequence (a_n)?

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5 Find all functions $f : Z \to Z$ such that $f(-1) = f(1)$ and $f(x)+f(y) = f(x+2xy)+f(y-2xy)$ for all $x, y \in Z$

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6 Show that there exists a tetrahedron which can be partitioned into eight congruent tetrahedra, each of which is similar to the original one.
