Czech And Slovak Mathematical Olympiad, Round III, Category A 1992

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by parmenides51

1 For a permutation $p(a_1, a_2, ..., a_{17})$ of $1, 2, ..., 17$, let k_p denote the largest k for which $a_1+...+a_k < a_{k+1}+...+a_{17}$. Find the maximum and minimum values of k_p and find the sum $\sum_p k_p$ over all permutations$p$.

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2 Let S be the total area of a tetrahedron whose edges have lengths a, b, c, d, e, f . Prove that $S \le \frac{\sqrt{3}}{6}(a^2 + b^2 + ... + f^2)$

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3 Let $S(n)$ denote the sum of digits of $n \in N$. Find all n such that $S(n) = S(2n) = S(3n) = ... = S(n^2)$

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4 Solve the equation $\cos 12x = 5 \sin 3x + 9\ tan^2 x +\ cot^2 x$

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5 The function $f : (0, 1) \to R$ is defined by $f(x) = x$ if x is irrational, $f(x) = \frac{p+1}{q}$ if $x = \frac{p}{q}$, where $(p, q) = 1$.
Find the maximum value of f on the interval $(7/8, 8/9)$.

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6 Let ABC be an acute triangle. The altitude from B meets the circle with diameter AC at points P, Q, and the altitude from C meets the circle with diameter AB at M, N. Prove that the points M, N, P, Q lie on a circle.

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