German National Olympiad 1965, Final Round

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by parmenides51

–	Day 1
1	For a given positive real parameter p, solve the equation $\sqrt{p+x}+\sqrt{p-x}=x$.
2	Determine which of the prime numbers $2, 3, 5, 7, 11, 13, 109, 151, 491$ divide $z = 1963^{1965} - 1963$.
3	Two parallelograms $ABCD$ and $A'B'C'D'$ are given in space. Points A'', B'', C'', D'' divide the segments AA', BB', CC', DD' in the same ratio. What can be said about the quadrilateral $A''B''C''D''$?
–	Day 2
4	Find the locus of points in the plane, the sum of whose distances from the sides of a regular polygon is five times the inradius of the pentagon.
5	Determine all triples of nonzero decimal digits (x, y, z) for which the equality $\sqrt{\underbrace{xxx...x}_{2n} - \underbrace{yy...y}_{n}} = \underbrace{zzz...z}_{n}$ holds for at least two different natural numbers n.
6	Let α, β, γ be the angles of a triangle. Prove that $\cos\alpha, + \cos\beta + \cos\gamma \le \frac{3}{2}$ and find the cases of equality.