**Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2000**

www.artofproblemsolving.com/community/c1079562

by parmenides51

**1a** Show that any odd number can be written as the difference between two perfect squares.

**1b** Determine if there is an infinite sequence $a_1, a_2, a_3, ..., a_n$ of positive integers such that for all $n \geq 1$ the sum $a_1^2 + a_2^2 + a_3^2 + ...^2 + a_n^2$ is a perfect square

**2a** Let $x, y$ and $z$ be real numbers such that $x + y + z = 0$. Show that $x^3 + y^3 + z^3 = 3xyz$.

**2b** Let $a, b, c$ and $d$ be non-negative real numbers such that $a + b + c + d = 4$.
Show that $\sqrt{a+b+c} + \sqrt{b+c+d} + \sqrt{c+d+a} + \sqrt{d+a+b} \geq 6$.

**3** a) Each point, on the perimeter of a square, is colored either red, or blue. Show that, there is a right-angled triangle where all the corners are on the square of the square and so that all the corners are on points of the same color.
b) Show that, it is possible to color each point on the perimeter of one square, red, white, or blue so that, there is not a right-angled triangle where all the three corners are at points of same color.

**4** For some values of c, the equation $x^c + y^c = z^c$ can be illustrated geometrically.
For example, the case $c = 2$ can be illustrated by a right-angled triangle. By this we mean that, x, y, z is a solution of the equation $x^2 + y^2 = z^2$ if and only if there exists a right-angled triangle with catheters $x$ and $y$ and hypotenuse $z$.
In this problem we will look at the cases $c = -\frac{1}{2}$ and $c = -1$.
a) Let $x, y$ and $z$ be the radii of three circles intersecting each other and a line, as shown, in the figure. Show that, $x^{-\frac{1}{2}} + y^{-\frac{1}{2}} = z^{-\frac{1}{2}}$
`https://cdn.artofproblemsolving.com/attachments/5/7/5315e33e1750a3a49ae11e1b5527311117ce7`
`png`
b) Draw a geometric figure that illustrates the case in a similar way, $c = -1$. The figure must be able to be constructed with a compass and a ruler. Describe such a construction and prove that, in the figure, lines $x, y$ and $z$ satisfy $x^{-1} + y^{-1} = z^{-1}$. (All positive solutions of this equation should be possible values for $x, y$, and $z$ on such a figure, but you don't have to prove that.)


© 2020 AoPS Incorporated

Art of Problem Solving is an ACS WASC Accredited School.