Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2001

www.artofproblemsolving.com/community/c1079567

by parmenides51

1a Suppose that a, b, c are real numbers such that $a+b+c>0$, and so the equation $ax^2+bx+c=0$ has no real solutions. Show that $c>0$.

1b Suppose that x and y are positive real numbers such that x^3, y^3 and $x+y$ are all rational numbers. Show that the numbers xy, x^2+y^2, x and y are also rational

---

2 Let A be a set, and let $P(A)$ be the powerset of all non-empty subsets of A. (For example, $A=\{1,2,3\}$, then $P(A)=\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$.)
A subset F of P (A) is called *strong* if the following is true:
If B_1 and B_2 are elements of F, then $B_1 \cup B_2$ is also an element of F.
Suppose that F and G are strong subsets of $P(A)$.
a) Is the union $F \cup G$ necessarily strong?
b) Is the intersection $F \cap G$ necessarily strong?

3a What is the largest possible area of a quadrilateral with sidelengths $1, 4, 7$ and 8 ?

3b The diagonals AC and BD in the convex quadrilateral $ABCD$ intersect in S. Let F_1 and F_2 be the areas of $\triangle\ ABS$ and $\triangle\ CSD$. and let F be the area of the quadrilateral $ABCD$. Show that $\sqrt{F_1}+\sqrt{F_2} \le \sqrt{F}$

---

4 At a two-day team competition in chess, three schools with 15 pupils each attend. Each student plays one game against each player on the other two teams, ie a total of 30 chess games per student.
a) Is it possible for each student to play exactly 15 games after the first day?
b) Show that it is possible for each student to play exactly 16 games after the first day.
c) Assume that each student has played exactly 16 games after the first day. Show that there are three students, one from each school, who have played their three parties

---