**Niels Henrik Abels Math Contest (Norwegian Math Olympiad) Final Round 2003**

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by parmenides51

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| **1a** | Let $x$ and $y$ are real numbers such that $x+y=2$ and $x^3+y^3=3$. What is $x^2+y^2$? |
| **1b** | Let $x_1, x_2, ..., x_n$ be real numbers in an interval $[m, M]$ such that $\sum_{i=1}^n x_i = 0$. Show that $\sum_{i=1}^n x_i^2 \le -nmM$ |
| **2a** | Find all pairs $(x, y)$ of integers numbers such that $y^3+5=x(y^2+2)$ |
| **2b** | Let $a_1, a_2, ..., a_n$ be $n$ different positive integers where $n \ge 1$. Show that $$\sum_{i=1}^n a_i^3 \ge \left(\sum_{i=1}^n a_i\right)^2$$ |
| **3** | Let $ABC$ be a triangle with $AC > BC$, and let $S$ be the circumscribed circle of the triangle. $AB$ divides $S$ into two arcs. Let $D$ be the midpoint of the arc containing $C$.<br>(a) Show that $\angle ACB + 2 \cdot \angle ACD = 180^o$.<br>(b) Let $E$ be the foot of the altitude from $D$ on $AC$. Show that $BC + CE = AE$. |
| **4a** | $25$ boys and $25$ girls sit around a table. Show that there is a person who has a girl sitting on either side of them. |
| **4b** | Let $m > 3$ be an integer. At a camp there are more than $m$ participants. The camp manager discovers that every time he picks out the camp participants, they say they have exactly one mutual friend among the participants. Which is the largest possible number of participants at the camp?<br>(If $A$ is a friend of $B$, $B$ is also a friend of $A$. A person is not considered a friend of himself.) |