German National Olympiad 2000, Final Round

www.artofproblemsolving.com/community/c1080074

by parmenides51

–	Day 1
1	For each real parameter a, find the number of real solutions to the system $\|x\|+\|y\|=1$, $x^2+y^2=a$.
2	For an integer $n \geq 2$, find all real numbers x for which the polynomial $f(x)=(x-1)^4+(x-2)^4+...+(x-n)^4$ takes its minimum value.
3	Suppose that an interior point O of a triangle ABC is such that the angles $\angle BAO, \angle CBO, \angle ACO$ are all greater than or equal to 30^o. Prove that the triangle ABC is equilateral.
–	Day 2
4	Find all nonnegative solutions (x,y,z) to the system $\sqrt{x+y}+\sqrt{z}=7$, $\sqrt{x+z}+\sqrt{y}=7$, $\sqrt{y+z}+\sqrt{x}=5$.
5	(a) Let be given $2n$ distinct points on a circumference, n of which are red and n are blue. Prove that one can join these points pairwise by n segments so that no two segments intersect and the endpoints of each segments have different colors. (b) Show that the statement from (a) remains valid if the points are in an arbitrary position in the plane so that no three of them are collinear.
6	A sequence (a_n) satisfies the following conditions: (i) For each $m \in N$ it holds that $a_{2^m}=1/m$. (ii) For each natural $n \geq 2$ it holds that $a_{2n-1}a_{2n}=a_n$. (iii) For all integers m,n with $2m>n\geq 1$ it holds that $a_{2n}a_{2n+1}=a_{2^m+n}$. Determine a_{2000}. You may assume that such a sequence exists.