German National Olympiad 2001, Final Round

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– Day 1

1 Determine all real numbers q for which the equation $x^4 - 40x^2 + q = 0$ has four real solutions which form an arithmetic progression

2 Determine the maximum possible number of points you can place in a rectangle with lengths 14 and 28 such that any two of those points are more than 10 apart from each other.

3 Wiebke and Stefan play the following game on a rectangular sheet of paper. They start with a rectangle with 60 rows and 40 columns and cut it in turns into smaller rectangles. The cuttings must be made along the gridlines, and a player in turn may cut only one smaller rectangle. By that, Stefan makes only vertical cuts, while Wiebke makes only horizontal cuts. A player who cannot make a regular move loses the game.
(a) Who has a winning strategy if Stefan makes the first move?
(b) Who has a winning strategy if Wiebke makes the first move?

– Day 2

4 In how many ways can the Nikolaus House (see the picture) be drawn? Edges may not be erased nor duplicated, and no additional edges may be drawn.
`https://cdn.artofproblemsolving.com/attachments/0/5/33795820e0335686b06255180af698e536a9b`
`png`

5 The Fibonacci sequence is given by $x_1 = x_2 = 1$ and $x_{k+2} = x_{k+1} + x_k$ for each $k \in N$.
(a) Prove that there are Fibonacci numbes that end in a 9 in the decimal system.
(b) Determine for which n can a Fibonacci number end in n 9-s in the decimal system.

6 (11) In a pyramid $SABCD$ with the base $ABCD$ the triangles ABD and BCD have equal areas. Points M, N, P, Q are the midpoints of the edges AB, AD, SC, SD respectively. Find the ratio between the volumes of the pyramids $SABCD$ and $MNPQ$

6 (12) Let ABC be a triangle with $\angle A = 90^o$ and $\angle B < \angle C$. The tangent at A to the circumcircle k of $\triangle\ ABC$ intersects line BC at D. Let E be the reflection of A in BC. Also, let X be the feet of the perpendicular from A to BE and let Y be the midpoint of AX. Line BY meets k again at Z. Prove that line BD is tangent to the circumcircle of $\triangle\ ADZ$.