Austria Beginners' Competition 2018

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by parmenides51

1 Let a, b and c denote positive real numbers. Prove that $\frac{a}{c}+\frac{c}{b}\geq\frac{4a}{a+b}$.
When does equality hold?

(Walther Janous)

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2 Let ABC be an acute-angled triangle, M the midpoint of the side AC and F the foot on AB of the altitude through the vertex C. Prove that $AM = AF$ holds if and only if $\angle BAC = 60^o$.

(Karl Czakler)

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3 For a given integer $n \ge 4$ we examine whether there exists a table with three rows and n columns which can be filled by the numbers $1, 2, ..., , 3n$ such that • each row totals to the same sum z and • each column totals to the same sum s.
Prove:
(a) If n is even, such a table does not exist.
(b) If $n = 5$, such a table does exist.

(Gerhard J. Woeginger)

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4 For a positive integer n we denote by $d(n)$ the number of positive divisors of n and by $s(n)$ the sum of these divisors. For example, $d(2018)$ is equal to 4 since 2018 has four divisors $(1, 2, 1009, 2018)$ and $s(2018) = 1 + 2 + 1009 + 2018 = 3030$.
Determine all positive integers x such that $s(x) \cdot d(x) = 96$.

(Richard Henner)
