Peru IMO TST 2019

www.artofproblemsolving.com/community/c1082745

by parmenides51, Jafet98

– Day 1

---

1 In each cell of a chessboard with 2 rows and 2019 columns a real number is written so that:

- There are no two numbers written in the first row that are equal to each other.
- The numbers written in the second row coincide with (in some another order) the numbers written in the first row.
- The two numbers written in each column are different and they add up to a rational number.

Determine the maximum quantity of irrational numbers that can be in the chessboard.

---

2 A *power* is a positive integer of the form a^k, where a and k are positive integers with $k \geq 2$. Let S be the set of positive integers which cannot be expressed as sum of two powers (for example, $4, 7, 15$ and 27 are elements of S). Determine whether the set S has a finite or infinite number of elements.

---

3 Let I, O and Γ be the incenter, circumcenter and the circumcircle of triangle ABC, respectively. Line AI meets Γ at M $(M \neq A)$. The circumference ω is tangent internally to Γ at T, and is tangent to the lines AB and AC. The tangents through A and T to Γ intersect at P. Lines PI and TM meet at Q. Prove that the lines QA and MO meet at a point on Γ.

– Day 2

---

4 Let $k \geq 0$ an integer. The sequence $a_0, a_1, a_2, a_3, \ldots$ is defined as follows:

- $a_0 = k$
- For $n \geq 1$, we have that a_n is the smallest integer greater than a_{n-1} so that $a_n + a_{n-1}$ is a perfect square.

Prove that there are exactly $\left\lfloor \sqrt{2k} \right\rfloor$ positive integers that cannot be written as the difference of two elements of such a sequence.
Note. If x is a real number, $\lfloor x \rfloor$ denotes the greatest integer smaller or equal than x.

---

5 Let m and n two given integers. Ana thinks of a pair of real numbers x, y and then she tells Beto the values of $x^m + y^m$ and $x^n + y^n$, in this order. Beto's goal is to determine the value of xy using that information. Find all values of m and n for which it is possible for Beto to fulfill his wish, whatever numbers that Ana had chosen.

© 2020 AoPS Incorporated

6 Let p and q two positive integers. Determine the greatest value of n for which there exists sets $A_1,\ A_2,\dots,\ A_n$ and $B_1,\ B_2,\dots,\ B_n$ such that:

- The sets $A_1,\ A_2,\dots,\ A_n$ have p elements each one.
- The sets $B_1,\ B_2,\dots,\ B_n$ have q elements each one.
- For all $1\le i,\ j\le n$, sets A_i and B_j are disjoint if and only if $i=j$.

AoPS Online AoPS Academy AoPS Beast ACADEMY

© 2020 AoPS Incorporated

Art of Problem Solving is an ACS WASC Accredited School.