Junior Olympiad of Malaysia 2015

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–	Day 1
1	Baron and Peter are playing a game. They are given a simple finite graph G with $n \geq 3$ vertex and k edges that connects the vertices. First Peter labels two vertices A and B, and places a counter at A. Baron starts first. A move for Baron is move the counter along an edge. Peter's move is to remove an edge from the graph. Baron wins if he reaches B, otherwise Peter wins. Given the value of n, what is the largest k so that Peter can always win?
2	Let $ABCD$ be a convex quadrilateral. Let angle bisectors of $\angle B$ and $\angle C$ intersect at E. Let AB intersect CD at F. Prove that if $AB+CD=BC$, then A, D, E, F is cyclic.
–	Day 2
3	Let a, b, c be positive real numbers greater or equal to 3. Prove that $$3(abc+b+2c) \geq 2(ab+2ac+3bc)$$ and determine all equality cases.
4	Given a natural number $n \geq 3$, determine all strictly increasing sequences $a_1 < a_2 < \cdots < a_n$ such that gcd$(a_1, a_2) = 1$ and for any pair of natural numbers (k, m) satisfy $n \geq m \geq 3$, $m \geq k$, $$\frac{a_1+a_2+\cdots+a_m}{a_k}$$ is a positive integer.
5	Navi and Ozna are playing a game where Ozna starts first and the two take turn making moves. A positive integer is written on the waord. A move is to (i) subtract any positive integer at most 2015 from it or (ii) given that the integer on the board is divisible by 2014, divide by 2014. The first person to make the integer 0 wins. To make Navi's condition worse, Ozna gets to pick integers a and b, $a \geq 2015$ such that all numbers of the form $an+b$ will not be the starting integer, where n is any positive integer. Find the minimum number of starting integer where Navi wins.