Kurschak Competition 2019

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by Ti-Ci

1   In an acute triangle $\triangle ABC$, $AB < AC < BC$, and A_1, B_1, C_1 are the projections of A, B, C to the corresponding sides. Let the reflection of B_1 wrt CC_1 be Q, and the reflection of C_1 wrt BB_1 be P.
Prove that the circumcirle of A_1PQ passes through the midpoint of BC.

2   Find all family $\mathcal{F}$ of subsets of $[n]$ such that for any nonempty subset $X \subseteq [n]$, exactly half of the elements $A \in \mathcal{F}$ satisfies that $|A \cap X|$ is even.

3   Is it true that if H and A are bounded subsets of $\mathbb{R}$, then there exists at most one set B such that $a + b (a \in A, b \in B)$ are pairwise distinct and $H = A + B$.