

Regional Competition For Advanced Students 2019

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- 1 Let x, y be real numbers such that $(x + 1)(y + 2) = 8$. Prove that

$$(xy - 10)^2 \geq 64.$$

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- 2 The convex pentagon $ABCDE$ is cyclic and $AB = BD$. Let point P be the intersection of the diagonals AC and BE . Let the straight lines BC and DE intersect at point Q . Prove that the straight line PQ is parallel to the diagonal AD .

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- 3 Let $n \geq 2$ be a natural number.
An $n \times n$ grid is drawn on a blackboard and each field with one of the numbers -1 or $+1$ labeled. Then the n row and also the n column sums calculated and the sum S_n of all these $2n$ sums determined.
(a) Show that for no odd number n there is a label with $S_n = 0$.
(b) Show that if n is an even number, there are at least six different labels with $S_n = 0$.

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- 4 Find all natural numbers n that are smaller than 128^{97} and have exactly 2019 divisors.
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