Federal Competition For Advanced Students, Part 2 2019

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–	Day 1
1	Determine all functions $f : R \to R$, such that $f(2x + f(y)) = x + y + f(x)$ for all $x, y \in R$. (Gerhard Kirchner)
2	A (convex) trapezoid $ABCD$ is good, if it is inscribed in a circle, sides AB and CD are the bases and CD is shorter than AB. For a good trapezoid $ABCD$ the following terms are defined: • The parallel to AD passing through B intersects the extension of side CD at point S. • The two tangents passing through S on the circumircle of the trapezoid touch the circle at E and F, where E lies on the same side of the straight line CD as A. Give the simplest possible equivalent condition (expressed in side lengths and / or angles of the trapezoid) so that with a good trapezoid $ABCD$ the two angles $\angle BSE$ and $\angle FSC$ have the same measure. (Walther Janous)
3	In Oddland there are stamps with values of 1 cent, 3 cents, 5 cents, etc., each for odd number there is exactly one stamp type. Oddland Post dictates: For two different values on a letter must be the number of stamps of the lower one value must be at least as large as the number of tokens of the higher value. In Squareland, on the other hand, there are stamps with values of 1 cent, 4 cents, 9 cents, etc. there is exactly one stamp type for each square number. Brands can be found in Squareland can be combined as required without further regulations. Prove for every positive integer n: there are the same number in the two countries possibilities to send a letter with stamps worth a total of n cents. It makes no difference if you have the same stamps on arrange a letter differently. (Stephan Wagner)
–	Day 2
4	Let a, b, c be the positive real numbers such that $a + b + c + 2 = abc$. Prove that $$(a + 1)(b + 1)(c + 1) \geq 27.$$
5	Let ABC be an acute-angled triangle. Let D and E be the feet of the altitudes on the sides BC or AC. Points F and G are located on the lines AD and BE in such a way that $\frac{AF}{FD} = \frac{BG}{GE}$. The

line passing through C and F intersects BE at point H, and the line passing through C and G intersects AD at point I. Prove that points F, G, H and I lie on a circle.

(Walther Janous)

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6 Find the smallest possible positive integer n with the following property:
For all positive integers x, y and z with $x|y^3$ and $y|z^3$ and $z|x^3$ always to be true that $xyz|(x+y+z)^n$.

(Gerhard J. Woeginger)
