geometry shortlists from Cono Sur Mathematical Olympiads, 1993, 2003, 2005, 2009, 2012, 2018, 2020 so far

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– 1993 additional problems

1993.1 Let C_1 and C_2 be two concentric circles and C_3 an outer circle to C_1 inner to C_2 and tangent to both. If the radius of C_2 is equal to 1, how much must the radius of C_1 be worth, so that the area of is twice that of C_3?

1993.2 Let $ABCD$ be a quadrilateral and let O be the point of intersection of diagonals AC and BD. Knowing that the area of triangle AOB is equal to 1, the area of triangle BOC is equal to 2, and the area of triangle COD is equal to 4, calculate the area of triangle AOD and prove that $ABCD$ is a trapezoid.

1993.3 Justify the following construction of the bisector of an angle with an inaccessible vertex: `https://cdn.artofproblemsolving.com/attachments/9/d/be4f7799d58a28cab3b4c515633b0e021c150` `png` $M \in a$ and $N \in b$ are taken, the 4 bisectors of the 4 internal angles formed by MN are traced with a and b. Said bisectors intersect at P and Q, then PQ is the bisector sought.

1993.4 Is it possible to locate in a rectangle of 5 cm by 8 cm, 51 circles of diameter 1 cm, so that they don't overlap? Could it be possible for more than 40 circles ?

1993.5 A block of houses is a square. There is a courtyard there in which a gold medal has fallen. Whoever calculates how long the side of said apple is, knowing that the distances from the medal to three consecutive corners of the apple are, respectively, 40 m, 60 m and 80 m, will win the medal.

1993.6 Consider in the interior of an equilateral triangle ABC points D, E and F such that$D$ belongs to segment BE, E belongs to segment CF and$F$ to segment AD. If $AD = BE = CF$ then DEF is equilateral.

1993.7 Let $ABCD$ be a convex quadrilateral, where M is the midpoint of DC, N is the midpoint of BC, and O is the intersection of diagonals AC and BD. Prove that O is the centroid of the triangle AMN if and only if $ABCD$ is a parallelogram.

1993.8 In a triangle ABC, let D, E and F be the touchpoints of the inscribed circle and the sides AB, BC and CA. Show that the triangles DEF and ABC are similar if and only if ABC is equilateral.

1993.9 Prove that a line that divides a triangle into two polygons of equal area and equal perimeter passes through the center of the circle inscribed in the triangle. Prove an analogous property for a polygon that has an inscribed circle.

1993.10 Let ω be the unit circle centered at the origin of R^2. Determine the largest possible value for the radius of the circle inscribed to the triangle OAP where P lies the circle and A is the projection of P on the axis OX.

1993.11 Let Γ be a semicircle with center O and diameter AB. D is the midpoint of arc AB. On the ray OD, we take E such that $OE = BD$. BE intersects the semicircle at F and P is the point on AB such that FP is perpendicular to AB. Prove that $BP = \frac{1}{3}AB$.

1993.12 Given 4 lines in the plane such that there are not 2 parallel to each other or no 3 concurrent, we consider the following 8 segments: in each line we have 2 consecutive segments determined by the intersections with the other three lines.
Prove that:
a) The lengths of the 8 segments cannot be the numbers $1, 2, 3, 4, 5, 6, 7, 8$ in some order.
b) The lengths of the 8 segments can be 8 different integers.

1993.13 Determine the real values of x such that the triangle with sides 5, 8, and x is obtuse.

1993.14 Prove that the sum of the squares of the distances from a point P to the vertices of a triangle ABC is minimum when P is the centroid of the triangle.

– 2003

2003.G1 Let O be the circumcenter of the isosceles triangle ABC ($AB = AC$). Let P be a point of the segment AO and Q the symmetric of P with respect to the midpoint of AB. If OQ cuts AB at K and the circle that passes through A, K and O cuts AC in L, show that $\angle ALP = \angle CLO$.

2003.G2 The circles C_1, C_2 and C_3 are externally tangent in pairs (each tangent to other two externally). Let M the common point of C_1 and C_2, N the common point of C_2 and C_3 and P the common point of C_3 and C_1. Let A be an arbitrary point of C_1. Line AM cuts C_2 in B, line BN cuts C_3 in C and line CP cuts C_1 in D. Prove that AD is diameter of C_1.

2003.G3 An interior P point to a square $ABCD$ is such that $PA = a, PB = b$ and $PC = b + c$, where the numbers a, b and c satisfy the relationship $a^2 = b^2 + c^2$. Prove that the angle BPC is right.

2003.G4 In a triangle ABC , let P be a point on its circumscribed circle (on the arc AC that does not contain B). Let H, H_1, H_2 and H_3 be the orthocenters of triangles ABC, BCP, ACP and ABP, respectively. Let $L = PB \cap AC$ and $J = HH_2 \cap H_1H_3$. If M and N are the midpoints of JH and LP, respectively, prove that MN and JL intersect at their midpoint.

2003.G5.4 In an acute triangle ABC, the points H, G, and M are located on BC in such a way that AH, AG, and AM are the height, angle bisector, and median of the triangle, respectively. It is known that $HG = GM$, $AB = 10$, and $AC = 14$. Find the area of triangle ABC.

2003.G6 Let L_1 and L_2 be two parallel lines and L_3 a line perpendicular to L_1 and L_2 at H and P, respectively. Points Q and R lie on L_1 such that $QR = PR$ ($Q \neq H$). Let d be the diameter of the circle inscribed in the triangle PQR. Point T lies L_2 in the same semiplane as Q with respect to line L_3 such that $\frac{1}{TH} = \frac{1}{d} - \frac{1}{PH}$. Let X be the intersection point of PQ and TH. Find the locus of the points X as Q varies on L_1.

2003.G7.3 Let ABC be an acute triangle such that $\angle B = 60$. The circle with diameter AC intersects the internal angle bisectors of A and C at the points M and N, respectively $(M \neq A, N \neq C)$. The internal bisector of $\angle B$ intersects MN and AC at the points R and S, respectively. Prove that $BR \leq RS$.

– 2005

2005.G1 Construct triangle given all lenght of it altitudes.

Please, do it elementary with Euclidian geometry (no trigonometry or coordinate geometry).

2005.G2 Find the ratio between the sum of the areas of the circles and the area of the fourth circle that are shown in the figure
Each circle passes through the center of the previous one and they are internally tangent.
`https://cdn.artofproblemsolving.com/attachments/d/2/29d2be270f7bcf9aee793b0b01c2ef10131e0`
`jpg`

2005.G3.4 Let ABC be a isosceles triangle, with $AB = AC$. A line r that pass through the incenter I of ABC touches the sides AB and AC at the points D and E, respectively. Let F and G be points on BC such that $BF = CE$ and $CG = BD$. Show that the angle $\angle FIG$ is constant when we vary the line r.

2005.G4.2 Let ABC be an acute-angled triangle and let AN, BM and CP the altitudes with respect to the sides BC, CA and AB, respectively. Let R, S be the pojections of N on the sides AB, CA, respectively, and let Q, W be the projections of N on the altitudes BM and CP, respectively.

(a) Show that R, Q, W, S are collinear.
(b) Show that $MP = RS - QW$.

2005.G5 Let O be the circumcenter of an acute triangle ABC and A_1 a point of the minor arc BC of the circle ABC . Let A_2 and A_3 be points on sides AB and AC respectively such that $\angle BA_1A_2 = \angle OAC$ and $\angle CA_1A_3 = \angle OAB$. Points B_2, B_3, C_2 and C_3 are similarly constructed, with B_2 in BC, B_3 in AB, C_2 in AC and C_3 in BC. Prove that lines A_2A_3, B_2B_3 and C_2C_3 are concurrent.

2005.G6 Let AM and AN be the tangents to a circle Γ drawn from a point A (M and N lie on the circle). A line passing through A cuts Γ at B and C, with B between A and C such that $AB : BC = 2 : 3$. If P is the intersection point of AB and MN, calculate the ratio AP : CP .

– 2009

2009.G1.6 Sebastian has a certain number of rectangles with areas that sum up to 3 and with side lengths all less than or equal to 1. Demonstrate that with each of these rectangles it is possible to cover a square with side 1 in such a way that the sides of the rectangles are parallel to the sides of the square.

Note: The rectangles can overlap and they can protrude over the sides of the square.

2009.G2 The trapezoid $ABCD$, of bases AB and CD, is inscribed in a circumference Γ. Let X a variable point of the arc AB of Γ that does not contain C or D. We denote Y to the point of intersection of AB and DX, and let Z be the point of the segment CX such that $\frac{XZ}{XC} = \frac{AY}{AB}$. Prove that the measure of $\angle AZX$ does not depend on the choice of X.

2009.G3 We have a convex polygon P in the plane and two points S, T in the boundary of P, dividing the perimeter in a proportion $1 : 2$. Three distinct points in the boundary, denoted by A, B, C start to move simultaneously along the boundary, in the same direction and with the same speed. Prove that there will be a moment in which one of the segments AB, BC, CA will have a length smaller or equal than ST.

2009.G4 Let AA_1 and CC_1 be altitudes of an acute triangle ABC. Let I and J be the incenters of the triangles AA_1C and AC_1C respectively. The C_1J and A_1I lines cut into T. Prove that lines AT and TC are perpendicular.

2009.G5.3 Let A, B, and C be three points such that B is the midpoint of segment AC and let P be a point such that $< PBC = 60$. Equilateral triangle PCQ is constructed such that B and Q are on different half=planes with respect to PC, and the equilateral triangle APR is constructed in such a way that B and R are in the same half-plane with respect to AP. Let X be the point of intersection of the lines BQ and PC, and let Y be the point of intersection of the lines BR and AP. Prove that XY and AC are parallel.

– 2012

2012.G1 Let $ABCD$ be a cyclic quadrilateral. Let P be the intersection of BC and AD. Line AC intersects the circumcircle of triangle BDP in points S and T, with S between A and C. Line BD intersects the circumcircle of triangle ACP in points U and V, with U between B and D. Prove that PS = PT = PU = PV.

2012.G2 Let ABC be a triangle, and M and N variable points on AB and AC respectively, such that both M and N do not lie on the vertices, and also, $AM \times MB = AN \times NC$. Prove that the perpendicular bisector of MN passes through a fixed point.

2012.G3 Let ABC be a triangle, and M, N, and P be the midpoints of AB, BC, and CA respectively, such that $MBNP$ is a parallelogram. Let R and S be the points in which the line MN intersects the circumcircle of ABC. Prove that AC is tangent to the circumcircle of triangle RPS.

2012.G4.2 2. In a square $ABCD$, let P be a point in the side CD, different from C and D. In the triangle ABP, the altitudes AQ and BR are drawn, and let S be the intersection point of lines CQ and DR. Show that $\angle ASB = 90$.

2012.G5 Let ABC be an acute triangle, and let H_A, H_B, and H_C be the feet of the altitudes relative to vertices A, B, and C, respectively. Define I_A, I_B, and I_C as the incenters of triangles AH_BH_C, BH_CH_A, and CH_AH_B, respectively. Let T_A, T_B, and T_C be the intersection of the incircle of triangle ABC with BC, CA, and AB, respectively. Prove that the triangles $I_AI_BI_C$ and $T_AT_BT_C$ are congruent.

2012.G6.6 6. Consider a triangle ABC with $1 < \frac{AB}{AC} < \frac{3}{2}$. Let M and N, respectively, be variable points of the sides AB and AC, different from A, such that $\frac{MB}{AC} - \frac{NC}{AB} = 1$. Show that circumcircle of triangle AMN pass through a fixed point different from A.

– 2018

2018.G1.1 Let $ABCD$ be a convex quadrilateral, where R and S are points in DC and AB, respectively, such that $AD = RC$ and $BC = SA$. Let P, Q and M be the midpoints of RD, BS and CA, respectively. If $\angle MPC + \angle MQA = 90$, prove that $ABCD$ is cyclic.

2018.G2.5 Let ABC be an acute-angled triangle with $\angle BAC = 60^\circ$ and with incenter I and circumcenter O. Let H be the point diametrically opposite(antipode) to O in the circumcircle of $\triangle BOC$. Prove that $IH = BI + IC$.

2018.G3 Consider the pentagon $ABCDE$ such that $AB = AE = x$, $AC = AD = y$, $\angle BAE = 90^o$ and $\angle ACB = \angle ADE = 135^o$. It is known that C and D are inside the triangle BAE. Determine the length of CD in terms of x and y.

2018.G4 Let ABC be an acute triangle with $AC > AB$. Let Γ be the circle circumscribed to the triangle ABC and D the midpoint of the smaller arc BC of this circle. Let I be the incenter of ABC and let E and F be points on sides AB and AC, respectively, such that $AE = AF$ and I lies on the segment EF. Let P be the second intersection point of the circumcircle of the triangle AEF with Γ with $P \neq A$. Let G and H be the intersection points of the lines PE and PF with Γ different from P, respectively. Let J and K be the intersection points of lines DG and DH with lines AB and AC, respectively. Show that the line JK passes through the midpoint of BC.

2018.G5 We say that a polygon P is inscribed in another polygon Q when all the vertices of P belong to the perimeter of Q. We also say in this case that Q is circumscribed to P. Given a triangle T, let ℓ be the largest side of a square inscribed in T and L is the shortest side of a square circumscribed to T . Find the smallest possible value of the ratio L/ℓ .

2018.G6 Let ABC be an acute triangle with circumcenter O and orthocenter H. The circle with center X_A passes through the points A and H and is tangent to the circumcircle of the triangle ABC. Similarly, define the points X_B and X_C. Let O_A, O_B and O_C be the reflections of O with respect to sides BC, CA and AB, respectively. Prove that the lines O_AX_A, O_BX_B and O_CX_C are concurrent.

– 2020

2020.G1.4 Let ABC be an acute scalene triangle. D and E are variable points in the half-lines AB and AC (with origin at A) such that the symmetric of A over DE lies on BC. Let P be the intersection of the circles with diameter AD and AE. Find the locus of P when varying the line segment DE.

2020.G2 Let ABC be a triangle whose inscribed circle is ω. Let r_1 be the line parallel to BC and tangent to ω, with $r_1 \neq BC$ and let r_2 be the line parallel to AB and tangent to ω with $r_2 \neq AB$. Suppose that the intersection point of r_1 and r_2 lies on the circumscribed circle of triangle ABC. Prove that the sidelengths of triangle ABC form an arithmetic progression.

2020.G3.3 Let ABC be an acute triangle such that $AC < BC$ and ω its circumcircle. M is the midpoint of BC. Points F and E are chosen in AB and BC, respectively, such that $AC = CF$ and $EB = EF$. The line AM intersects ω in $D \neq A$. The line DE intersects the line FM in G. Prove that G lies on ω.

2020.G4 Let ABC be a triangle with circumcircle ω. The bisector of $\angle BAC$ intersects ω at point A_1. Let A_2 be a point on the segment AA_1, CA_2 cuts AB and ω at points C_1 and C_2, respectively. Similarly, BA_2 cuts AC and ω at points B_1 and B_2, respectively. Let M be the intersection point of B_1C_2 and B_2C_1. Prove that MA_2 passes the midpoint of BC.

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2020.G5 posted as 2021 p6

– 2021

2021.G1.2 Let ABC be a triangle and I its incenter. The lines BI and CI intersect the circumcircle of ABC again at M and N, respectively. Let C_1 and C_2 be the circumferences of diameters NI and MI, respectively. The circle C_1 intersects AB at P and Q, and the circle C_2 intersects AC at R and S. Show that P, Q, R and S are concyclic.

2021.G2 Let ABC be an acute triangle. Define A_1 the midpoint of the largest arc BC of the circumcircle of ABC . Let A_2 and A_3 be the feet of the perpendiculars from A_1 on the lines AB and AC , respectively. Define B_1, B_2, B_3, C_1, C_2, and C_3 analogously. Show that the lines A_2A_3, B_2B_3, C_2C_3 are concurrent.

2021.G3 Let $ABCD$ be a parallelogram with vertices in order clockwise and let E be the intersection of its diagonals. The circle of diameter DE intersects the segment AD at L and EC at H. The circumscribed circle of LEB intersects the segment BC at O. Prove that the lines HD , LE and BC are concurrent if and only if $EO = EC$.

2021.G4 Let ABC be a triangle and Γ the A- exscribed circle whose center is J . Let D and E be the touchpoints of Γ with the lines AB and AC, respectively. Let S be the area of the quadrilateral $ADJE$, Find the maximum value that $\frac{S}{AJ^2}$ has and when equality holds.

2021.G5 Let $\triangle ABC$ be a triangle with circumcenter O, orthocenter H, and circumcircle ω. AA', BB' and CC' are altitudes of $\triangle ABC$ with A' in BC, B' in AC and C' in AB. P is a point on the segment AA'. The perpenicular line to $B'C'$ from P intersects BC at K. AA' intersects ω at $M \neq A$. The lines MK and AO intersect at Q. Prove that $\angle CBQ = \angle PBA$.

2021.G6.6 Let ABC be a scalene triangle with circle Γ. Let P, Q, R, S distinct points on the BC side, in that order, such that $\angle BAP = \angle CAS$ and $\angle BAQ = \angle CAR$. Let U, V, W, Z be the intersections, distinct from A, of the AP, AQ, AR and AS with Γ, respectively. Let $X = UQ \cap SW$, $Y = PV \cap ZR$, $T = UR \cap VS$ and $K = PW \cap ZQ$. Suppose that the points M and N are well determined, such that $M = KX \cap TY$ and $N = TX \cap KY$. Show that M, N, A are collinear.

2021.G7 Given an triangle ABC isosceles at the vertex A, let P and Q be the touchpoints with AB and AC, respectively with the circle T, which is tangent to both and is internally tangent to the circumcircle of ABC. Let R and S be the points of the circumscribed circle of ABC such that $AP = AR = AS$. Prove that RS is tangent to T .