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by Invert\_DOG\_about\_centre\_0

– Spring 2018 A-level Junior

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1. Thirty nine nonzero numbers are written in a row. The sum of any two neighbouring numbers is positive, while the sum of all the numbers is negative. Is the product of all these numbers negative or positive? (4 points)

Boris Frenkin

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2. Aladdin has several gold ingots, and sometimes he asks the Genie to give him more. The Genie first adds a thousand ingots, but then takes half of the total number. Could it be possible that after asking the Genie for gold ten times, the number of Aladdins gold ingots increased, assuming that each time the Genie took half, he took an integer number of ingots? (5 points)

Alexandr Perepechko

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3. Do there exist 2018 positive irreducible fractions, each with a different denominator, so that the denominator of the difference of any two (after reducing the fraction) is less than the denominator of any of the initial 2018 fractions? (6 points)

Maxim Didin

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4. Let  $O$  be the center of the circumscribed circle of the triangle  $ABC$ . Let  $AH$  be the altitude in this triangle, and let  $P$  be the base of the perpendicular drawn from point  $A$  to the line  $CO$ . Prove that the line  $HP$  passes through the midpoint of the side  $AB$ . (6 points)

Egor Bakaev

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5. There are 100 houses in the street, divided into 50 pairs. In each pair houses are right across the street one from another. On the right side of the street the houses have even numbers, while the houses on the left side have odd numbers. On both sides of the street the numbers increase from the beginning to the end of the street, but are not necessarily consecutive (some numbers may be omitted). For each house on the right side of the street, the difference between its number and the number of the opposite house was computed, and it turned out that all these values were different. Let  $n$  be the greatest number of a house on this street. Find the smallest possible value of  $n$ . (8 points)

Maxim Didin

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6. In the land of knights (who always tell the truth) and liars (who always lie), 10 people sit at a

round table, each at a vertex of an inscribed regular 10-gon, at least one of them is a liar. A traveler can stand at any point outside the table and ask the people: What is the distance from me to the nearest liar at the table? After that each person at the table gives him an answer. What is the minimal number of questions the traveler has to ask to determine which people at the table are liars? (Both the people at the table and the traveler should be considered as points, and everyone can compute the distance between any two points) (10 points)

Maxim Didin

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7. You are in a strange land and you don't know the language. You know that ! and ? stand for addition and subtraction, but you don't know which is which. Each of these two symbols can be written between two arguments, but for subtraction you don't know if the left argument is subtracted from the right or vice versa. So, for instance,  $a?b$  could mean any of  $a - b$ ,  $b - a$ , and  $a + b$ . You don't know how to write any numbers, but variables and parenthesis work as usual. Given two arguments  $a$  and  $b$ , how can you write an expression that equals  $20a - 18b$ ? (12 points)

Nikolay Belukhov

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