Czech-Polish-Slovak Junior Match 2014

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by parmenides51

–	Individual
1	On the plane circles k and ℓ are intersected at points C and D, where circle k passes through the center L of circle ℓ. The straight line passing through point D intersects circles k and ℓ for the second time at points A and B respectively in such a way that D is the interior point of segment AB. Show that $AB = AC$.
2	Solve the equation $a + b + 4 = 4\sqrt{a\sqrt{b}}$ in real numbers
3	We have 10 identical tiles as shown. The tiles can be rotated, but not flipper over. A 7×7 board should be covered with these tiles so that exactly one unit square is covered by two tiles and all other fields by one tile. Designate all unit sqaures that can be covered with two tiles. `https://cdn.artofproblemsolving.com/attachments/d/5/6602a5c9e99126bd656f997dee3657348d98b png`
4	The number a_n is formed by writing in succession, without spaces, the numbers $1, 2, ..., n$ (for example, $a_{11} = 1234567891011$). Find the smallest number t such that $11 \| a_t$.
5	A square is given. Lines divide it into n polygons. What is he the largest possible sum of the internal angles of all polygons?
–	Team
1	The set of $\{1, 2, 3, ..., 63\}$ was divided into three non-empty disjoint sets A, B. Let a, b, c be the product of all numbers in each set A, B, C respectively and finally we have determined the greatest common divisor of these three products. What was the biggest result we could get?
2	Let $ABCD$ be a parallelogram with $\angle BAD < 90^o$ and $AB > BC$. The angle bisector of BAD intersects line CD at point P and line BC at point Q. Prove that the center of the circle circumscribed around the triangle CPQ is equidistant from points B and D.
3	Find with all integers n when $\|n^3 - 4n^2 + 3n - 35\|$ and $\|n^2 + 4n + 8\|$ are prime numbers.
4	Point M is the midpoint of the side AB of an acute triangle ABC. Circle with center M passing through point C, intersects lines AC, BC for the second time at points P, Q respectively. Point R lies on segment AB such that the triangles APR and BQR have equal areas. Prove that lines PQ and CR are perpendicular.

5 There is the number 1 on the board at the beginning. If the number a is written on the board, then we can also write a natural number b such that $a + b + 1$ is a divisor of $a^2 + b^2 + 1$. Can any positive integer appear on the board after a certain time? Justify your answer.

6 Determine the largest and smallest fractions $F = \frac{y-x}{x+4y}$
if the real numbers x and y satisfy the equation $x^2y^2 + xy + 1 = 3y^2$.