Czech-Polish-Slovak Junior Match 2015

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by parmenides51

–	Individual
1	In the right triangle ABC with shorter side AC the hypotenuse AB has length 12. Denote T its centroid and D the feet of altitude from the vertex C. Determine the size of its inner angle at the vertex B for which the triangle DTC has the greatest possible area.
2	Decide if the vertices of a regular 30-gon can be numbered by numbers $1, 2, .., 30$ in such a way that the sum of the numbers of every two neighboring to be a square of a certain natural number.
3	Real numbers x, y satisfy the inequality $x^2 + y^2 \le 2$. Orove that $xy + 3 \ge 2x + 2y$
4	Let ABC ne a right triangle with $\angle ACB = 90^o$. Let E, F be respecitvely the midpoints of the BC, AC and CD be it's altitude. Next, let P be the intersection of the internal angle bisector from A and the line EF. Prove that P is the center of the circle inscribed in the triangle CDE .
5	Determine all natural numbers$n > 1$ with the property: For each divisor $d > 1$ of number n, then $d - 1$ is a divisor of $n - 1$.
–	Team
1	Let I be the center of the circle of the inscribed triangle ABC and M be the center of its side BC. If $\|AI\| = \|MI\|$, prove that there are two of the sides of triangle ABC, of which one is twice of the other.
2	We removed the middle square of 2×2 from the 8×8 board. a) How many checkers can be placed on the remaining 60 boxes so that there are no two not jeopardize? b) How many at least checkers can be placed on the board so that they are at risk all 60 squares? (A lady is threatening the box she stands on, as well as any box she can get to in one move without going over any of the four removed boxes.)
3	Different points A and D are on the same side of the line BC, with $\|AB\| = \|BC\| = \|CD\|$ and lines AD and BC are perpendicular. Let E be the intersection point of lines AD and BC. Prove that $\|\|BE\| - \|CE\|\| < \|AD\|\sqrt{3}$

4 Determine all such pairs pf positive integers (a, b) such that $a + b + (gcd(a, b))^2 = lcm(a, b) = 2 \cdot lcm(a - 1, b)$, where $lcm(a, b)$ denotes the smallest common multiple, and $gcd(a, b)$ denotes the greatest common divisor of numbers a, b.

5 Find the smallest real constant p for which the inequality holds $\sqrt{ab} - \frac{2ab}{a+b} \le p\left(\frac{a+b}{2} - \sqrt{ab}\right)$ with any positive real numbers a, b.

6 The vertices of the cube are assigned $1, 2, 3..., 8$ and then each edge we assign the product of the numbers assigned to its two extreme points. Determine the greatest possible the value of the sum of the numbers assigned to all twelve edges of the cube.

