Canada National Olympiad 2020

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by SpecialBeing2017

1 There are $n \geq 3$ distinct positive real numbers. Show that there are at most $n-2$ different integer power of three that can be written as the sum of three distinct elements from these n numbers.

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2 $ABCD$ is a fixed rhombus. Segment PQ is tangent to the inscribed circle of $ABCD$, where P is on side AB, Q is on side AD. Show that, when segment PQ is moving, the area of ΔCPQ is a constant.

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3 There are finite many coins in Davids purse. The values of these coins are pair wisely distinct positive integers. Is that possible to make such a purse, such that David has exactly 2020 different ways to select the coins in his purse and the sum of these selected coins is 2020?

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4 $S = \{1, 4, 8, 9, 16, ...\}$is the set of perfect integer power. ($S = \{n^k | n, k \in Z, k \geq 2\}$.)We arrange the elements in S into an increasing sequence $\{a_i\}$. Show that there are infinite many n, such that $9999 | a_{n+1} - a_n$

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5 Simple graph G has 19998 vertices. For any subgraph $\bar{G}$ of G with 9999 vertices, $\bar{G}$ has at least 9999 edges. Find the minimum number of edges in G
