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**Thailand Mathematical Olympiad 2016**

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by parmenides51, reveryu

– Day 1

**1** Let $ABC$ be a triangle with $AB \neq AC$. Let the angle bisector of $\angle BAC$ intersects $BC$ at $P$ and intersects the perpendicular bisector of segment $BC$ at $Q$. Prove that $\frac{PQ}{AQ} = \left(\frac{BC}{AB+AC}\right)^2$

**2** Let $M$ be a positive integer, and $A = \{1, 2, ..., M+1\}$. Show that if $f$ is a bijection from $A$ to $A$ then $\sum_{n=1}^{M} \frac{1}{f(n)+f(n+1)} > \frac{M}{M+3}$

**3** Determine all functions $f : R \to R$ satisfying $f(f(x)f(y) + f(y)f(z) + f(z)f(x)) = f(x) + f(y) + f(z)$ for all real numbers $x, y, z$.

**4** Each point on the plane is colored either red, green, or blue. Prove that there exists an isosceles triangle whose vertices all have the same color.

**5** given $p_1, p_2, ...$ be a sequence of integer and $p_1 = 2$,
for positive integer $n$, $p_{n+1}$ is the least prime factor of $np_1^{1!}p_2^{2!}...p_n^{n!} + 1$
prove that all primes appear in the sequence

(Proposed by Beatmania)

– Day 2

**6** Let $m$ and $n$ be positive integers. Prove that if $m^{4^n+1} - 1$ is a prime number, then there exists an integer $t \geq 0$ such that $n = 2^t$.

**7** Given $P(x) = a_{2016}x^{2016} + a_{2015}x^{2015} + ... + a_1x + a_0$
be a polynomial with real coefficients and $a_{2016} \neq 0$
satisfies

$|a_1 + a_3 + ... + a_{2015}| > |a_0 + a_2 + ... + a_{2016}|$

Prove that $P(x)$ has an odd number of complex roots with absolute value less than $1$ (count multiple roots also)

edited: complex roots

**8** Let $\triangle\ ABC$ be an acute triangle with incenter $I$. The line passing through $I$ parallel to $AC$ intersects $AB$ at $M$, and the line passing through $I$ parallel to $AB$ intersects $AC$ at $N$. Let the line $MN$ intersect the circumcircle of $\triangle ABC$ at $X$ and $Y$ . Let $Z$ be the midpoint of arc $BC$ (not containing $A$). Prove that $I$ is the orthocenter of $\triangle XYZ$

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**9** A real number $a \neq 0$ is given. Determine all functions $f : R \to R$ satisfying $f(x)f(y)+f(x+y) = axy$ for all real numbers $x, y$.

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**10** A *Pattano coin* is a coin which has a blue side and a yellow side. A positive integer not exceeding $100$ is written on each side of every coin (the sides may have different integers).
Two Pattano coins are *identical* if the number on the blue side of both coins are equal and the number on the yellow side of both coins are equal.
Two Pattano coins are *pairable* if the number on the blue side of both coins are equal or the number on the yellow side of both coins are equal.
Given $2559$ Pattano coins such that no two coins are identical. Show that at least one Pattano coin is pairable with at least $50$ other coins

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